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Control allocation for aircraft with input constraints based on improved cuckoo search algorithm

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ABSTRACT

The control allocation problem of aircraft whose control inputs contain integer constraints is investigated. The control allocation problem is described as an integer programming problem and solved by the cuckoo search algorithm. In order to enhance the search capability of the cuckoo search algorithm, the adaptive detection probability and amplification factor are designed. Finally, the control allocation method based on the proposed improved cuckoo search algorithm is applied to the tracking control problem of the innovative control effector aircraft. The comparative simulation results demonstrate the superiority and effectiveness of the proposed improved cuckoo search algorithm in control allocation of aircraft.

1. Introduction

As the development of the aircraft technology, for superior maneuver and reliability, more and more advanced aircraft, even unmanned aerial vehicles and missiles, deploy multiple and redundant effectors on their bodies. Therefore, an appropriate control allocation method is necessary for the control systems of these aircraft to use their effectors efficiently. Control allocation is a hot issue in the field of flight control. Many methods have been proposed for solving various control allocation problems [1]. Most of the time, the control allocation problem can be represented as an optimization problem. Therefore, the designed control allocation methods are usually based on some optimization methods.

For an aircraft, its control allocation problem is apparently related to its effectors. Sometimes some unconventional effectors will make the control allocation problem different from the conventional problems and difficult to be solved. A characteristic example is the innovative control effector (ICE) aircraft which is introduced by Lockheed Martin Tactical Aircraft Systems [2]. This aircraft uses several distributed arrays each of which contains lots of actuators as effectors. The particularity is that each actuator can only provide either full or no control energy [3]. We can group those actuators which have almost the same full control energies together and then the control allocation problem of the ICE aircraft can be translated into an integer programming problem with some constraints. Actually, the integer constraints are present in many aircraft effectors, such as the reaction control system (RCS) and so on.

Different from the normal linear programming or quadratic programming problem, most of the integer programming problems are non-deterministic polynomial hard (NP-hard) problems and their optimum solutions are usually hard to be obtained. Some classical methods, such as the branch-bound method and cutting plane method, are usually used to solve the integer linear programming problem and they are effective when the scale of the problem is small. However, with the increase of the scale of the problem, the computational complexities of the classical methods will increase rapidly and cannot meet the practical requirements. Therefore, the metaheuristic algorithms have attracted more and more attentions. There have been recently many studies using metaheuristic algorithms to solve aircraft control allocation problems [4–7]. The cuckoo search algorithm (CSA) is a relatively novel and promising metaheuristic algorithm proposed by Yang and Deb [8]. Some studies have demonstrated that its search capability is better than many other metaheuristic algorithms [9,10]. Therefore, the CSA has been applied to many application domains, such as

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parallel machine scheduling [11], total cost of ownership for supplier selection problem [12], maximum power point tracking for Photovoltaic System [13] and structural damage identification [14]. However, the efficiency of the basic CSA is still unsatisfactory. Therefore, the algorithm need be improved when it is used.

In this paper, an improved CSA is proposed for solving the aircraft control allocation problem with integer constraints. The remaining sections of this paper are organized as follows. Section 2 describes the aircraft control allocation problem with integer constraints. Section 3 formulates the design of the control allocation method based on an improved CSA. The simulation results established upon the proposed method and some compared methods are given in Section 4. Finally, some concluding remarks are summarized in Section 5.

2. Problem formulation

2.1. Description of the aircraft model

Consider the linearized dynamic model of aircraft

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}_v\boldsymbol{v}(t) \tag{1}$$

where $\boldsymbol{x}\in\mathrm{R}^{c_1}$ represents the states and $\boldsymbol{v}\in\mathrm{R}^{c_2}$ represents the virtual control input. The control objective is to track a reference model

$$\dot{\boldsymbol{x}}_{\mathrm{m}}(t) = \boldsymbol{A}_{\mathrm{m}}\boldsymbol{x}_{\mathrm{m}}(t) + \boldsymbol{B}_{\mathrm{m}}r(t) \tag{2}$$

where $r(t)$ is the reference input. Assume that the system (A, B_v) is known and controllable, then the virtual control law can be designed as

$$\boldsymbol{v}(t) = -\boldsymbol{K}_1\boldsymbol{x}(t) + \boldsymbol{K}_2r(t) \tag{3}$$

where $\boldsymbol{K}_1, \boldsymbol{K}_2$ are the gain matrices which meet the following matched conditions

$$\begin{cases} \boldsymbol{A} - \boldsymbol{B}_v\boldsymbol{K}_1 = \boldsymbol{A}_{\mathrm{m}} \\ \boldsymbol{B}_v\boldsymbol{K}_2 = \boldsymbol{B}_{\mathrm{m}} \end{cases} \tag{4}$$

Let $\boldsymbol{e}$ represent the tracking error. Under the virtual control law (3) the tracking error dynamical system can be described by

$$\dot{\boldsymbol{e}}(t) = \dot{\boldsymbol{x}}(t) - \dot{\boldsymbol{x}}_{\mathrm{m}}(t) = \boldsymbol{A}_{\mathrm{m}}\boldsymbol{e}(t) \tag{5}$$

The system (5) shows that the tracking error $\boldsymbol{e}$ is asymptotically stable provided $\boldsymbol{A}_{\mathrm{m}}$ is a Hurwitz matrix.

2.2. Control allocation problem

After obtaining the virtual control input $\boldsymbol{v}$, the control system on the aircraft need deploy the effectors to achieve it. Let $\boldsymbol{n} = [n_1, \ldots, n_{c_3}]^{\mathrm{T}}$ represent the actual control command from the effectors. Assume that $n_i, i = 1, 2, \ldots, c_3$ is restricted to the following compact set Ω_i

$$\Omega_i = \{n_i | \underline{n}_i \le n_i \le \overline{n}_i, n_i \in Z\} \tag{6}$$

and the effector model is a linear model in the form

$$\boldsymbol{v} = \boldsymbol{B}_u\boldsymbol{n}, n_i \in \Omega_i \tag{7}$$

The objective of control allocation is to find out a suitable actual control command $\boldsymbol{n}$ to achieve the virtual control input $\boldsymbol{v}$. Then the problem can be converted into an integer programming problem and the fitness function is defined as

$$J(\boldsymbol{n}) = \|\boldsymbol{C}(\boldsymbol{v} - \boldsymbol{B}_u\boldsymbol{n})\|_2, \quad n_i \in \Omega_i \tag{8}$$

where $\boldsymbol{C}$ is a definite weighting matrix, $\|\bullet\|_2$ is 2-norm.

It should be noticed that we cannot obtain the actual control command by rounding $\boldsymbol{n} = \boldsymbol{B}_u^{+}\boldsymbol{v}$ where $(\bullet)^{+}$ represents pseudo-inverse because the solution is inexact. Next, we propose an improved CSA for the design of the control allocation method.

3. Design of control allocation method

3.1. Basic cuckoo search algorithm

The CSA combines the cuckoo breeding behavior with a random walk called Lévy flight. Some studies have demonstrated that Lévy flight is an optimized search pattern for non-replenishable targets at unknown positions [15,16]. The CSA is established based on the following three idealized rules [8]:

1) Each cuckoo lays only one egg once and places it in a random nest.
2) The best nest including the high quality egg will be carried over to the next generation.
3) The egg can be discovered by the nest owner with a detection probability $p_a \in [0, 1]$. When one egg is discovered, the nest owner will abandon the nest and build a new one in somewhere.

Controlled by the detection probability p_a and a random number θ drawn from a uniform distribution $\theta \sim U(0, 1)$, the CSA uses a combination of local random walk and global explorative random walk to search the optimum solution. Let $\boldsymbol{\tau}_j^g$ represent the j-th cuckoo in the population of g-th generation. When generating a new solution $\boldsymbol{\tau}_j^{g+1}$, if $p_a \ge \theta$, the global random walk is performed via Lévy flights

$$\boldsymbol{\tau}_j^{g+1} = \boldsymbol{\tau}_j^g + \boldsymbol{\alpha} \otimes \boldsymbol{L}(\lambda) \tag{9}$$

where $\alpha > 0$ is the amplification factor, $\otimes$ denotes Hadamard multiplication, $\boldsymbol{L}(\lambda) = [l_1(\lambda), \cdots, l_{c_3}(\lambda)]^{\mathrm{T}}$ is the random step path and $l_i(\lambda)$ follows the Lévy distribution

$$l_i(\lambda) \sim g^{-\lambda}, \quad (1 < \lambda \le 3) \tag{10}$$

where λ is a Lévy flight parameter, $l_i(\lambda)$ can be obtained by

$$l_i(\lambda) = \frac{\mu}{|\nu|^{1/\beta}} \tag{11}$$

where $\beta = \lambda - 1$, μ and ν are drawn from normal distributions

$$\mu \sim N\left(0, \sigma_\mu^2\right), \quad \nu \sim N\left(0, \sigma_\nu^2\right) \tag{12}$$

where σ_μ, σ_ν represent the standard deviations of the corresponding normal distributions and the values are

$$\sigma_\mu = \left\{\frac{\Gamma(1+\beta)\sin(\pi\beta/2)}{\Gamma[(1+\beta)/2]\beta 2^{(\beta-1)/2}}\right\}^{1/\beta}, \quad \sigma_\nu = 1 \tag{13}$$

where Γ is the standard Gamma function.

If $p_a < \theta$, the new solution is produced by local random walk which can be written as

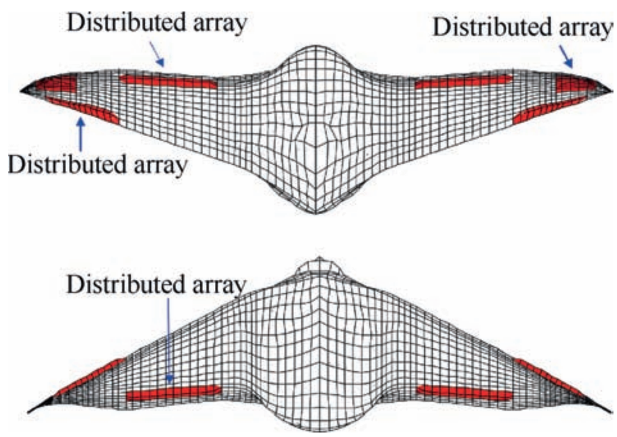

Fig. 1. The configuration of ICE aircraft.

$$\boldsymbol{\tau}_j^{g+1} = \boldsymbol{\tau}_j^g + \boldsymbol{\alpha}\otimes \boldsymbol{H}\left(p_a - \overline{\theta}\right)\otimes\left(\boldsymbol{\tau}_{j1}^g - \boldsymbol{\tau}_{j2}^g\right) \tag{14}$$

where $\boldsymbol{H}(\bullet)$ is the Heaviside function, $\overline{\theta}\sim U(0,1)$ is a random number, $\boldsymbol{\tau}_{j1}^g$ and $\boldsymbol{\tau}_{j2}^g$ are two different randomly chosen solutions.

3.2. Improved cuckoo search algorithm

Similar to the other metaheuristic algorithms, the process of the CSA is of randomicity. It can be observed from the generative mechanism of the solutions of CSA that the characteristic of updated population is controlled by two parameters, detection probability and amplification factor. However, the two parameters are aptotic in basic CSA; this form is open-loop and cannot reflect the dynamic of the search process. Therefore, it is necessary for CSA to introduce some dynamic adjustment mechanisms to make the parameters of the algorithm adaptable to the specific situation of the search process. The specific improvements are designed as follows.

1) Detection probability

Generally, during the initial phase of the search process, the global search is preferred for extensive exploration and during the final phase the local search is preferred for rapid convergence. On the other hand, it is proved that larger detection probability is good for global search and smaller detection probability is good for local search. Thus, according to the Rechenberg's 1/5 criteria which considered that the successful proportion of all the mutation actions should be 1/5 [17], the following adaptive method is adopted towards the detection probability

$$p_a = 0.05 + 0.3g/g_{\max} \tag{15}$$

where g is the present iterative number, $g_{\max}$ is the maximum number of iterations. Under the designed dynamic detection probability, the mutation rate in the whole searching process is about 0.2 on average.

2) Amplification factor

The amplification factor can adjust the length of the random step path produced via Lévy flights. Due to the differences among the nests, the amplification factor of each nest should be adaptively adjusted according to its quality. The low-quality nest should be assigned a large amplification factor for searching high-quality areas and the high-quality nest should be assigned a small amplification factor for searching the optimum solution around its location. Thus, the following adaptive method is adopted towards the amplification factor

$$\boldsymbol{\alpha}_j^g = \begin{cases} 2.5\overline{\boldsymbol{\alpha}}, & J\left(\boldsymbol{n}_j^g\right)\geq 5J_{\text{best}}^g \\ \left(0.5J\left(\boldsymbol{n}_j^g\right)\Big/J_{\text{best}}^g\right)\overline{\boldsymbol{\alpha}}, & \textit{others} \end{cases} \tag{16}$$

where $\overline{\boldsymbol{\alpha}}$ is a given reference amplification factor, J_{best}^t is the best fitness value in t-th generation.

3.3. Bounds of the nests

The new produced nests from (9) may not meet the constraints (6). For this reason, when a new nest $\boldsymbol{n}^* = [n_1^*, n_2^*, n_3^*, n_4^*, n_5^*, n_6^*]^{\mathrm{T}}$ is produced, the following measure is adopted to amend it

$$\hat{n}_i^* = \begin{cases} \overline{n}_i & , \ n_i^* > \overline{n}_i \\ \underline{n}_i & , \ n_i^* < \underline{n}_i \\ n_i^* & , \ \textit{others} \end{cases} \tag{17}$$

Then the amended nest $\hat{\boldsymbol{n}}^* = round([\hat{n}_1^*, \hat{n}_2^*, \hat{n}_3^*, \hat{n}_4^*, \hat{n}_5^*, \hat{n}_6^*]^{\mathrm{T}})$ where $round(\bullet)$ denotes the rounding approximation is an appropriate solution.

4. Simulation

In this section, the effectiveness of the proposed improved CSA in aircraft control allocation is verified. The control allocation method based on the proposed algorithm is applied to the ICE aircraft linearized lateral-directional model which can be described via Eq. (1). The states $\boldsymbol{x}\in\mathrm{R}^{C_4}$ can be defined as $\boldsymbol{x} = [V\ p\ r\ \phi]^{\mathrm{T}}$ where V represents the body-axis lateral velocity, p, r and ϕ represent the body-axis roll rate, yaw rate and roll angle, respectively. The configuration is illustrated in Fig. 1.

The parameters of the ICE aircraft model are given as follows [6,7]

$$\boldsymbol{A} = \begin{bmatrix} -0.0134 & 48.5474 & -632.3724 & 32.0756 \\ -0.0199 & -0.1209 & 0.1628 & 0 \\ -0.0024 & -0.0526 & -0.0252 & 0 \\ 0 & 1 & 0.0768 & 0 \end{bmatrix}, \quad \boldsymbol{B}_v = \begin{bmatrix} 0 & 0 \\ -0.0431 & 0.0476 \\ -0.0076 & -0.0023 \\ 0 & 0 \end{bmatrix}$$

Table 1
The bounds of the nests.

Boundary value	n_1	n_2	n_3	n_4	n_5	n_6
$\underline{n}_i$	−22	−22	−5	−5	−12	−12
$\overline{n}_i$	22	22	5	5	12	12

Table 2
The parameters of the improved CSA.

Parameter	Value	Meaning
ζ	20	number of nests
$g_{\max}$	100	maximum number of iterations
$\overline{\boldsymbol{\alpha}}$	$[0.6, 0.6, 0.15, 0.15, 0.3, 0.3]^{\mathrm{T}}$	reference amplification factor
λ	2.5	Lévy flight parameter

$$\boldsymbol{K}_1 = \begin{bmatrix} 9.9 & 63.6 & -1278.8 & -11.9 \\ -2.8 & 92.3 & -653.8 & 91.2 \end{bmatrix}, \quad \boldsymbol{K}_2 = \begin{bmatrix} -4.9903 \\ 16.4898 \end{bmatrix}$$

The reference model and reference input are [6,7].

Table 3
The statistical results based on two algorithms.

Algorithm	Mean fitness value	Best fitness value	Best actual command
Improved CSA	0.0105	7.4e-4	$[18,-15,-5,4,-1,2]^T$
Basic CSA	0.0132	0.0014	$[11,-12,5,4,7,-9]^T$

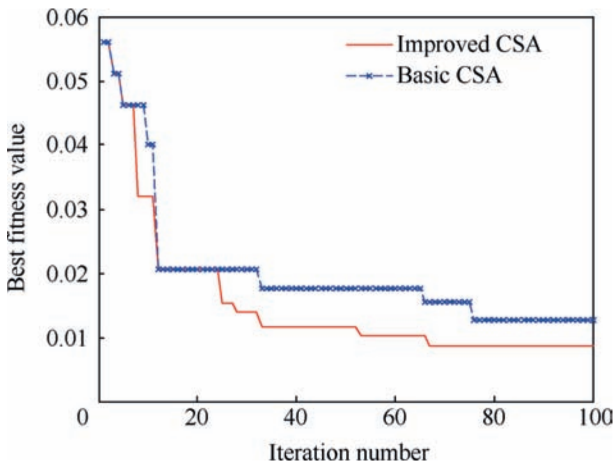

Fig. 2. The optimization convergence processes based on two algorithms.

$$\boldsymbol{A}_\mathrm{m} = \begin{bmatrix} -0.0134 & 48.5474 & -632.3724 & 32.0756 \\ 0.5386 & -1.7746 & -23.8313 & -4.8526 \\ 0.0664 & 0.6431 & -11.2476 & 0.1192 \\ 0 & 1 & 0.0768 & 0 \end{bmatrix}$$

$$\boldsymbol{B}_\mathrm{m} = [0\ 1\ 0\ 0]^\mathrm{T}$$

$$r(t) = \begin{cases} 0.4 & 10i \le t \le 10i+5, i = 0,1,2,\cdots \\ -0.4 & others \end{cases}$$

The matrix $\boldsymbol{B}_u$ and definite weighting matrix $\boldsymbol{C}$ are

$$\boldsymbol{B}_u = \begin{bmatrix} 0.65 & 0 & 0.2991 & 0.3205 & -1.6025 & -1.4568 \\ 0 & 0.6 & -0.3189 & -0.3416 & -1.8069 & -1.6426 \end{bmatrix}$$

$$\boldsymbol{C} = \begin{bmatrix} -0.1658 & -0.2207 \\ 0 & 0.0001 \\ -0.0004 & -0.0014 \\ -0.0076 & -0.0275 \end{bmatrix}$$

The bounds of nests are given in Table 1.

The parameters of the improved CSA are shown in Table 2.

First, the performance of the proposed improved CSA is studied. The individual optimization problem (8) is solved by the proposed algorithm. For comparison, the basic CSA is introduced to solve the same problem. To eliminate the difference of each experiment, the improved CSA and basic CSA use the same initial population and random walks produced by Lévy flights and they are executed 50 times. The virtual control input is selected to be $\boldsymbol{v} = [7, -6]^\mathrm{T}$. The comparative results are shown in Table 3.

It can be seen that both of the mean and best fitness values

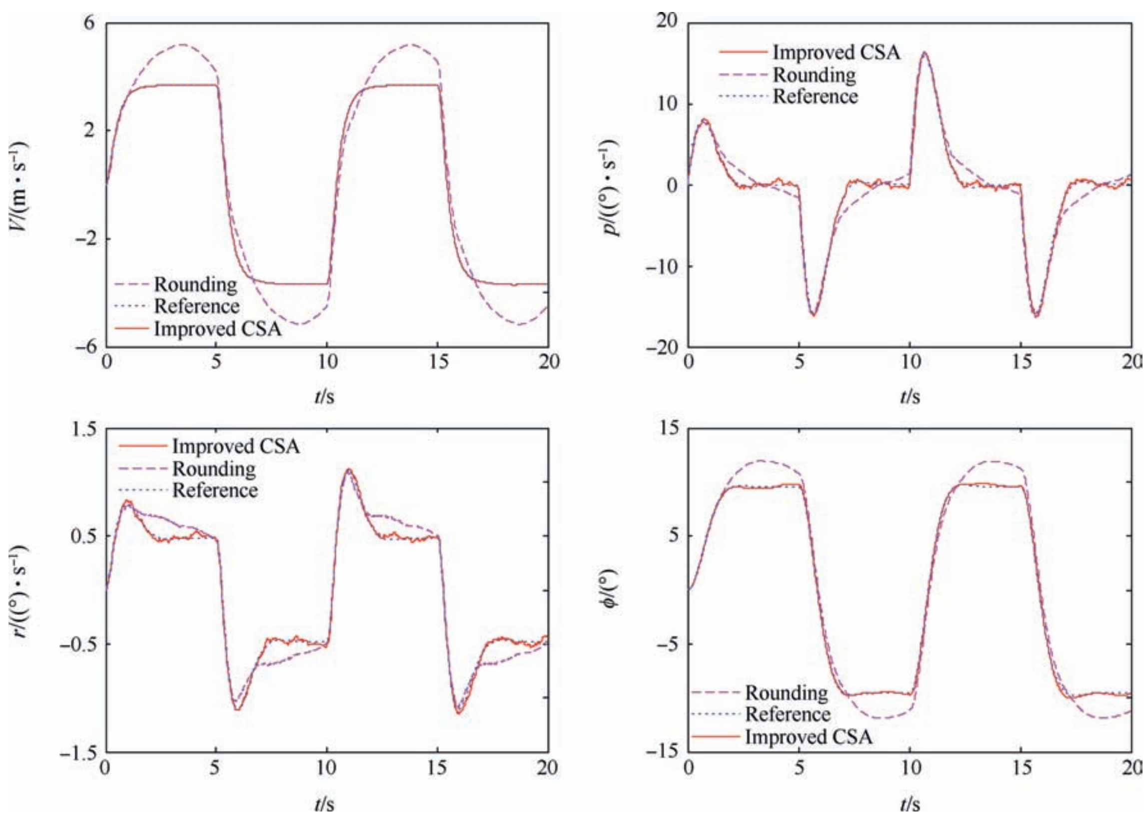

Fig. 3. The state responses of the closed-loop system.

under the proposed improved CSA are better than the basic CSA. Fig. 2 shows the typical search processes under the two algorithms. It can be observed that the search capability of the proposed improved CSA is better than the basic CSA. According to the comparative results it can be concluded that the proposed improved CSA is superior to the basic CSA.

Next, the flight process of the ICE aircraft is studied. The simulation time and fixed-step size are chosen to be 20 s and 0.02 s respectively. For comparison, the method of direct rounding from [18] is introduced. The contrastive actual commands are obtained via the equation $\boldsymbol{n} = round(\boldsymbol{B}_u^{+}\boldsymbol{v})$. The comparative results are illustrated in Fig. 3.

It can be observed that the tracking errors of the system states under the direct rounding method are obvious and the control allocation method based on the proposed improved CSA achieves better tracking performance. It can be concluded that the proposed control allocation method is necessary and effective for the tracking control of the ICE aircraft.

5. Conclusions

In this paper, an improved cuckoo search algorithm is proposed for solving the aircraft control allocation problem with integer constraints. Different from the basic cuckoo search algorithm with fixed parameters, the improved algorithm uses adaptive parameters varying according to the search progress and quality of each nest to enhance the algorithm search capability. The flight control system uses the improved algorithm to obtain the actual control commands. The ICE aircraft model is introduced for verifying the effectiveness of the proposed method. The comparative simulation results show that the proposed improved cuckoo search algorithm is superior to the basic algorithm and the tracking performance of the aircraft control system under the control allocation method based on the proposed algorithm is quite better than the contrastive method.

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